Bean There, Done That: Computer-Assisted Design of Bean Sculptures

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Chicago's status as a world-class city is cemented by its iconic bean sculpture. Other cities, wanting to replicate the success, have muddied the bean waters by introducing their own bean variations: New York City has a bean sharing similar properties, and Ottawa has a sphere, dubbed the "Ottawa bean" by locals. Our economic analysis proves their worth, so naturally other cities will want their own. We present a mathematical model of the space of all bean sculptures, and an algorithm to help cities replace existing landmarks with beans.

FIG. 1: Michaelangelo's *David* improved by being turned into a bean sculpture (photo by Wikimedia Commons Korido, CC BY-SA 4.0)

I. INTRODUCTION

Public art is important: It can serve as an expression of culture, heritage, and creativity within a community. It has the power to stimulate dialogue and provoke thought. It is a nice thing to go look at with one's friends[§], or to walk past on one's way to work. On top of this, successful public art installations can contribute to the economic vitality of a city by attracting tourists, fostering a sense of place, and enhancing the overall appeal of urban environments. Indeed, successful public art can become an icon for its host city, putting it on the map in a way that is not otherwise possible.

Consider as an example "The Bean" (known to nerds and um ackchuyually types as "Cloud Gate"), a sculpture by artist Anish Kapoor that was unveiled in Chicago in 2006 [1]. Almost 20 years after its construction, the Bean stands as a symbol of Chicago and a success story of public art. Having observed the Bean's impact (perhaps even feeling threatened by it) New York City, a city with no dearth of art and culture, commissioned another bean structure from Kapoor, which was completed in 2023. Naturally one begins to suspect that we are seeing the beginnings of a revolution within the art world and public life more broadly.

As the saying goes, twice is a coincidence, but three times is a pattern: Recently, a bean-like structure has been rediscovered by locals in Ottawa, Ontario. The piece was built in 1966 and originally called "The Sphere" [10], but it has recently enjoyed a surge in popularity since its rebranding as "The Ottawa Bean". Though less curvy than Chicago's, Ottawa bean is still a bean. In fact, it is the simplest bean (that is, the trivial bean), the result of taking away all possible bends and dimples.

The growing success of this third reflective bean cements the potential of bean structures to revolutionize public art. It is important to note that the Ottawa Bean

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§This is true even if one's friends misrepresent the nature of the public art to which one is being led.

FIG. 2: Beans in Chicago, New York City, and Ottawa, respectively. NYC photo ©Bracht Bug, CC BY-NC-ND 2.0.

is not by any particularly famous artist†. This point solidifies the intuition that the reflectivity and bean morphology are the key factors in the beans' appeal, rather than some arcane feature of Anish Kapoor's style in particular. This begs the question: How far can we take this? Can other communities emulate the success of these three beans? And how can one create a bean for one's own city?

One possibility would be to commission artists, but communities may be deterred from this option due to artists' sometimes difficult personalities, and their insistence on being paid for their work. A more pragmatic option would be to automate the design of the beans; this is a realistic possibility, due to the beans' simple forms. Although one could randomize the bean parameters, this kind of approach could be criticized for not actually being art, due to a lack of inspiration or underlying meaning. We propose an approach in which beans are designed based on other objects; in particular, here we base our designs off iconic landmarks, with the intention of eventually replacing them with beans to maximize the impact of the new beans.

In Section II we present some general background to motivate the construction of more bean sculptures. In Section III we outline a mathematical model for beans that can neatly describe existing beans, but is flexible enough to accommodate a variety of new beans in a similar style. We accomplish this by formulating beans as a signed distance function for the smooth union of one or more quadratic Bézier curves. The existing and agreed-upon bean sculptures fit nicely into the parameter space ("bean-space") as single curves, while compound beans can reproduce structures visually similar to other city landmarks while still appearing definitively like a bean. To help along the bean-hopeful, in Section IV we provide an algorithm to automatically fit existing city structures into their nearest bean-space equivalent. In Section V we apply the model to some examples, and we conclude in Section VI.

†The authors mean no offense to Art Price, progenitor of the Ottawa Bean, but he does not have a Wikipedia page and that's just a fact.

II. BACKGROUND

A. Why Beans?

We begin with a brief economic argument for prioritizing the construction of bean sculptures. The first notable bean, the one in Chicago, provides ample reason for other cities to want to follow suit. It is situated in Millennium Park. Prior to construction of the bean, the park had a total of *zero* annual visitors. After construction began, the city estimated it had 5 million annual visitors. [5] That's an incalculable increase from 0. Figure 3 shows how, in literature, references to Millennium Park went through the roof after construction of the bean began. In the first half of 2016, the city counted 12,859,360 visitors—approximately 26 million visitors annually—making it the "#1 attraction in the Midwest and among the top 10 most-visited sites in the U.S." [5]

That's a lot of visitors, but how does it stack up against other individual attractions? As a point of comparison, we look at Ottawa's Parliament Hill. In a 2007 report, it had just 3 million annual visitors. [4] A recent restoration project for Parliament's Centre Block is budgeted as a \$4.5-5 billion project [8]. Chicago's bean, meanwhile, costed just \$23 million. [2]

We crunched the visitor numbers, and came to the following conclusion:

$$\frac{5{,}000{,}000}{3{,}000{,}000} > 1 \tag{1}$$

We also crunched the numbers for the cost:

$$\frac{\$23{,}000{,}000}{\$4{,}500{,}000{,}000} < 1 \tag{2}$$

The natural conclusion that Ottawa would come to is that it would be more financially responsible to replace Parliament with a bean. As it is only a matter of time before other cities come to this conclusion, too, we provide an algorithm to replace something like Parliament with a bean for minimal disruption to the existing space.

B. Why Replace Landmarks?

We understand that cities may feel incentivized to simply build a new bean in a new location rather than replace existing landmarks. The main argument against this is an environmental one. A single bean will likely not satiate cities. If cities were to keep expanding every time they want a new bean, it would contribute to an unprecedented level of urban sprawl that is simply irresponsible in our present environmental crisis.

The other main reason to replace existing landmarks instead is because leaving landmarks in place impedes inevitable progress. We know that everything is chrome in the future [9] and replacing landmarks with beans is a clear path to that future.

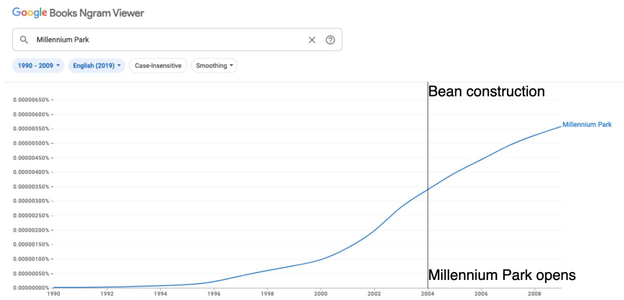

FIG. 3: Google Ngram Viewer data on references to Millennium Park in literature. Interest in Millennium Park skyrocketed in the leadup to and the introduction of the bean.

III. BEAN-SPACE MODEL

We use a quadratic Bézier as the base component of a bean since it is able to capture both the bent shape of the Chicago bean, but also in the trivial case where all control points are zero, the spherical Ottawa bean.

A single segment is unable to capture the potential range of all future beans, so we allow beans to be composed of multiple segments. Simply taking the union of multiple segments appears rigid and decidedly unbeanlike (see Figure 5, left), so we instead define the surface in a way that allows us to use a *smooth* union (see Figure 5, right.) Signed Distance Functions allow such a smooth union to be defined succinctly, so we chose this format to represent our bean surfaces. This means that we represent the surface of a bean as the isosurface $f(X) = 0$, where f describes the signed distance to the surface. At a high level, f describes the smooth union between multiple quadratic Bézier segments.

A. Signed Distance Functions

A signed distance function (SDF) is a function $f : \mathbb{R}^n \mapsto \mathbb{R}$ describing the distance to a surface at a given point in space. Mapping out the isosurface of $f(X) = 0$ yields the surface described by the function. This can be done via the Marching Cubes algorithm to produce a 3D mesh, or via sphere tracing to produce an image.

SDFs are a flexible surface representation if one wants to organically join multiple base shapes. While taking $u(d_1, d_2) = \min\{d_1, d_2\}$ of two surfaces produces a surface representing the union of the shapes, the *smooth union* operation $u(d_1, d_2) = d_1 + kg(d_2 - d_1)/k)$ blends smoothly between its two inputs when they are a distance of k apart, using the kernel g to control the curve of the blending.

FIG. 4: Examples of different quadratic Bézier segments with our bend constraints.

FIG. 5: Different values for the smoothness k between segments: 0, 0.05, and 0.2, respectively.

B. Formulation

Each quadratic Bézier segment is referenced in f via a function representing the signed distance to the centerline of the segment, $Q(X; C)$. [6] Here, $C \in R^{3\times3}$ describes the three control points to the function. For brevity, we omit the full definition of Q. We subtract a radius r from B to give the segment thickness.

In practice, we constrain C such that $C_{i,1} = A_i$, $C_{i,3} = B_i$, and $C_{i,2} = (A_i + B_i)/2 + b\hat{n}$, where $b \in \mathbb{R}$ and $\hat{n}$ is a normalized vector such that $\hat{n}\dot{(}C_3 - C_1) = 0$. In effect, the middle control point is always halfway between the first and last control point, plus an offset normal to the line between them. This ensures a physically plausible curve with no self-intersections. Examples of segments fitting these constraints are shown in Figure 4.

We combine each segment using the *SDF smooth union operator* $U(d_1, d_2; k)$, which blends the distance between two input surface distances when they are a distance k or less away: [7]

$$U(d_1, d_2; k) = d_1 + kg(d_2 - d_1)/k \tag{3}$$

Figure 5 shows the effect of the smoothness parameter k on the final surface.

Using the above, we represent the bean surface f recursively: the final value $f(X; n)$ is defined as the smooth union of the n^{th} segment with $f(X; n-1)$, with the base case being a single segment:

$$f(X; i) = \begin{cases} Q(X; C_0) - r_0, & i = 1 \\ U(B(X; C_i) - r_i, f(X; i-1), k), & i > 1 \end{cases} \tag{4}$$

To summarize, a bean in f has the following parameters:

- n, the number of Bézier segments

- r_i, $0 < i \leq n$, the radius of each segment
- A_i, $0 < i \leq n$, the start point for each segment
- B_i, $0 < i \leq n$, the end point for each segment
- b_i, $0 < i \leq n$, the amount of bend in each segment
- k, the smoothing between segments

IV. BEAN OF BEST FIT

Given an target image, we want to optimize our bean-space parameters to find a bean that best matches the target. The target image T is a 300×300-pixel one-bit image representing a mask of the space we want a bean to occupy. We define a function $P(n, r, A, B, b, k; T)$ that defines the score of a set of parameters in relation to the target image T. Using the bean-space parameters, we render similarly-sized image M of the bean they represent. We then define P as the intersection-over-union between the pixel grids M and T. Maximizing this number encourages maximum coverage of the target area with minimal overlap with areas we don't want covered:

$$P(M;T) = \frac{\sum_{i,j} M_{i,j} \wedge T_{i,j}}{\sum_{i,j} M_{i,j} \vee T_{i,j}} \tag{5}$$

To perform the optimization, we need to use a method of optimization that does not rely exclusively on gradients, as the parameter n is an integer and therefore does not have a useful gradient. We pick the Metropolis-Hastings algorithm to explore the space defined by our bean parameters as a probabilistic, gradient-free optimization algorithm.

A. Metropolis-Hastings Optimization

Markov Chain Monte Carlo (MCMC) algorithms are algorithms used to sample probability distributions $P(x)$ that are difficult to sample directly. In our case, bean-space, weighted by similarity to an input image, is such a difficult distribution.

The Metropolis-Hastings Algorithm is one such MCMC algorithm. Starting with one sample x (in our case, a set of bean parameters), a new candidate sample x' is selected from an easier-to-sample distribution, $Q(x'|x)$, such as by randomly mutating x. A random number $u \in [0, 1]$ is generated: if $u < P(x')/P(x)$, the candidate sample is accepted; otherwise, it is rejected. Intuitively, a sample where $P(x)$ is higher than the current sample is always accepted, but there is still a chance of acceptance of lower-valued samples, too, allowing jumps into other areas of the distribution. This property is useful for exploring highly non-convex spaces.

To use this procedure for optimization, $P(x)$ can be treated as as function to optimize, and a record of the value of x that produced the highest-seen value of $P(x)$ can be kept. This is effectively a zero-gradient optimization, since we only evaluate $P(x)$, not $dP(x)/dx$. This can be useful for mixed-integer optimization problems where a gradient would not exist. It also has the added benefit of being simple enough that the authors can implement it themselves, and not pay for an expensive license for optimization software. [3]

V. RESULTS AND ANALYSIS

A. Validation

We found the bean of best fit of known beans in order to validate the bean-fitting algorithm. The expected result would be a bean matching the shape of the input bean, as it is already a bean and does not need to be rebeaned. Both should only include one Bèzier segment, and further, the Ottawa bean should have all its control points equal to 0 to form a sphere. Figure 6 shows the result of the optimization process when run on the Chicago and Ottawa beans, where it successfully matches the shape of the input.

FIG. 6: Existing beans recreated via our bean-fitting algorithm: Chicago's bean (top) and Ottawa's bean (bottom).

Now that we know our bean-fitting algorithm works on existing beans, we explore its impact on replacing structures yet to be beaned.

Since the purpose of beaning a city is to place it on the map, it is natural that cities would want to ensure its bean is in a significant, easy-to-access location. A logical place to start, then, would be to replace a city's most notable landmarks with an equivalent bean. Figure 7 shows some examples of notable Ottawa landmarks: the Parliament buildings are maybe the motivation for most grade school field trips to Ottawa, but are unlikely to be a stand-out attraction in the eyes of the students visiting. A bean replacement will increase tourist satisfaction with no downsides.

FIG. 7: Beaned Ottawa landmarks: *Maman* by Louise Bourgeois (photo by John Talbot, CC BY 2.0), and the Parliament buildings.

B. Novel Inputs

Figure 8 shows yet more examples of Canadian and world landmarks replaced by beans. The beans effectively retain the form and character of the originals. The BN Tower maintains the same erect stature as the CN Tower (Figure 8, first row.) In Pisa, tourists can still pose and hold up the Beaning Tower the same way they would have held up the Leaning Tower (Figure 8, second row.) We believe this tight integration with the environment is sure to increase bean acceptance and tourism.

VI. CONCLUSION

Having established the economic advantage of replacing tired monuments with bean sculptures, we have presented an algorithm for designing bean sculptures on the basis of these existing landmarks: We establish a model of bean-space by creating smooth-unioned ensembles of simple beans, each parametrized by a quadratic Bézier curve. The parameters of the hyper-bean are then optimized to achieve the best fit to a given landmark. We apply our model to some example landmarks and we find good results, even with a relatively low-dimensional bean space. These results can be improved upon simply by increasing the number of parameters used in defining the hyper-bean; the cost of computing time for this should be small compared to the cost of hiring actual people to design beans.

FIG. 8: Improved world landmarks: the CN Tower (photo by Wikimedia Commons user Wladyslaw, CC BY-SA 3.0), the Leaning Tower of Pisa (photo by Wikimedia Commons user U3207458, CC BY-SA 4.0), the Great Pyramid of Giza (photo by Wikimedia Commons user Nina, CC BY 2.5), and the Earth (photo by NASA/Bill Anders)

We also envision extending this model to create beans for other purposes, beyond public art installations and/or upgrades to existing infrastructure and landmarks. For example, there may be a market for smaller personal beans, or even custom beans based on objects with some personal significance or sentimental value.

But let not the motivation for the proliferation of beans be only capitalistic. As a society, we need beans. Their reflective surface inspires us to take a close look at ourselves. Their smooth contours and cold surfaces are soothing, a much needed balm in the feverish times of this 21st century. In the warped surface of a bean, one sees one's surroundings distorted, and one is inspired to see the world from a new perspective. Our world of right angles and matte surfaces has gotten us this far, but to progress further as a species, it is crucial that we embrace a radically different attitude in the decoration of our public and private spaces. The model we have presented in this work will without a doubt be an important tool in this effort of beanification.

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